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UNSYMMETRICAL FORCES IN AN AIRPLANE CELL

By R. Vogt

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Summary

This paper calls attention to the desirability of expanding airplane building regulations to include proof of safety for cases of unsymmetrical loading, at least in the structural members which are thereby specially stressed. These flight cases involve increases of the customary load assumptions through rudder deflection (yawing moment) and aileron deflection (rolling moment). Corresponding increases in the magnitude of these moments can be found on the basis of wind-tunnel tests or theoretical considerations.

As shown by an example, the stresses are particularly great in the case of a one-sided landing shock, because the principal part of the total inertia moment of an airplane lies in the wings. With respect to this case, it would be very advisable to make provision for the transmission of the stresses not only to the fuselage, but also to the real absorption points, namely, the wings.

Most countries with airplane industries of any account have enacted regulations regarding the stresses which an airplane

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*"Unsymmetrische Kräfte in der Flugzeugzelle," from Zeitschrift für Flugtechnik und Motorluftschiffahrt, June 14, 1929, pp.274-276.

cell must be able to withstand. The loading conditions assumed in all these regulations agree quite well in principle and differ as regards the magnitude of the stresses only in so far as the uses of the airplanes differ.

Since these regulations are based both on practical experience and on theoretical considerations, they are not to be regarded as final, but as susceptible to improvement in the light of further experience. The following observations and calculations are to be considered in this sense, i.e., as suggestions.

With the exception of a single special case, these regulations assume the wing loading to be symmetrical. From this it might be inferred that all the unsymmetrical loads are included in the required assumptions for the determination of the load factors, so that, for such an exceptional case, the factor of safety would be somewhat smaller than it is in symmetrical loading. We could agree with such a conception if, in unsymmetrical loading, the safety factor of all structural members would be reduced in the same proportion as expressed by the load factor. This is not the case, however. If we take, for example, a continuous spar with three points of support, it is easily conceivable that the middle support receives very little or no load, which is often the actual case. Hence the building regulations require members leading from this support to be designed for only very small stresses.

Unsymmetrical stresses in the cell, even of a relatively small order of magnitude, produce, however, such large additional stresses in these members as to eliminate all question of airworthiness. Hence it seems desirable to obtain some idea of the possible stresses in cases of unsymmetrical loading. We can imagine the following cases in which such stresses occur.

The deflection of the rudder produces a moment about the vertical axis of an airplane. The magnitude of this moment is known and is used in combination with the moment of the horizontal tail surfaces in calculating the fuselage. This moment affects the union of the cell with the fuselage in so far as a considerable part of the inertia moment of the whole airplane about its vertical axis resides in the wing. It would be advisable to introduce into the calculation the breaking load of the vertical tail surfaces for this moment and to assign such a proportion of it to the wing as the inertia moment of the wing bears to the inertia moment of the whole airplane.

The deflection of the ailerons in banking produces a rolling moment about the longitudinal axis of an airplane. The magnitude of this moment can be determined most accurately by means of wind-tunnel tests or, with sufficient accuracy for the constructor, by means of aerodynamic calculations such as have been published on several occasions in this magazine. On the basis of these calculations and in the absence of wind-tunnel tests the writer has adopted another method for finding the rolling

moment. A record was made of the time required by a skillful pilot to make a whole or half roll. If uniform acceleration be assumed (up to a quarter roll, for example), the stresses produced in the cell can then be calculated.

In utilizing this rolling moment, however determined, we must consider how it is balanced, The larger part of the inertia moment about the longitudinal axis resides in the wing tips just where the moment is produced. Its transmission to the fuselage is governed only by the ratio of the inertia moment of the latter. For a biplane with ailerons it would be necessary to determine only the additional forces on the upper wings which must be transmitted to the lower wings. The union between the upper wing and the fuselage could be seriously involved only in the case of a biplane with cantilever wings with ailerons on but one wing, because in this case the share of the total inertia moment residing in the lower wing must be taken into account as well as the share residing in the fuselage. In this connection attention is called to the favorable arrangement of a cantilever biplane with torsion struts near the wing tips.

It is obvious in any case that the magnitude of the possible stresses depends entirely on the design of the airplane. Though the forces in question are negligible in many cases, they may nevertheless become important in unfavorable arrangements. Hence it is well to require the calculation to be made with reference to the leveling off (e.g., after a spiral dive).

Another kind of unsymmetrical stressing may result from a one-sided landing. The calculation of this case is usually required by the regulations but, in my opinion, too little attention is paid to the fact that it is not sufficient, in this case of unsymmetrical loading, to test only the main landing-gear supports which are directly involved. The shock in a one-sided landing is in exactly the opposite direction to the aileron stresses in flight. Just because the chief component of the total inertia moment about the longitudinal axis resides in the wing, the chief component of the turning shock must also be transmitted to the wing. In order to show the danger of this case, we will make a mathematical investigation of a seaplane with two floats.

We shall base the calculation on the following reasonable values: span 20 m (65.6 ft.); wing area 74 m² (796.5 sq.ft.); distance between floats 5 m (16.4 ft.; total weight 4000 kg (8818 lb.); weight of wing 800 kg (1764 lb.) (Fig. 1). The distance of the center of gravity from the middle line of the wing is represented by h. Then a mass element m dx of the wing at the distance x from the plane of symmetry is subjected to a force

$$dP = r\,\omega\,m\,dx$$

where ω is the angular acceleration and r the radial distance from the center of gravity S (Fig. 2). The moment about the

center of gravity is

$$d\,M_f = r\,dP = r^2\,\omega\,m\,dx$$
$$= (h^2 + x^2)\,\omega\,m\,dx.$$

The total wing moment is then

$$M_f = 2\,m\,\omega \int_0^{b/2} (h^2 + x^2)\,dx$$
$$= 2\,m\,\omega \left| x\,h^2 + \frac{x^3}{3} \right|_0^{b/2}$$
$$= m\,b\,\omega\left(h^2 + \frac{b^2}{12}\right).$$

Now $m\,b$ is the wing mass, so that, with the numerical values of our example, we have

$$M_f = \frac{700}{9.81}\,\omega\left(1.5^2 + \frac{20^2}{12}\right)$$
$$= 2540\,\omega.$$

If we assume the shock of a one-sided landing to be three times the weight, the generated moment will be

$$M_l = 2.5 \times 3G = 2.5 \times 3 \times 4000$$
$$= 30000 \text{ mkg}.$$

The wing's share of the total inertia moment (calculated at 70%, though it is often still more) is

$$M_l' = 0.7 \times 30000 = 21000 \text{ mkg}.$$

The angular acceleration of the airplane now becomes

$$\omega = \frac{M_l'}{M_f} = \frac{21000}{2540} = 8.27\ s^{-2}$$

We are now in a position to calculate the resulting lateral

stresses in the cell. They consist of two components obtained from the integration of the horizontal components dH of the peripheral forces dP and from the horizontal components produced by the reaction of the points of support on the wing struts. The former and smaller component is

$$dH = dP \cos \alpha_x = r\ \omega\ m\ dx\ \frac{h}{r}$$

$$= \omega\ m\ h\ dx$$

$$H = m\ \omega\ h \int_{-b/2}^{+b/2} dx$$

$$= m\ b\ \omega\ h.$$

Now $m\ b$ is the mass of the wing $\omega\ h$ is the horizontal lateral component of the motion of the wing mass considered as concentrated in the middle. We find

$$H = \frac{700}{9.81} \times 8.27 \times 1.5 = 887 \text{ kg}$$

The reaction R of the point of support on the wing strut, located at a distance of 6 m (19.7 ft.) from the middle, is found from the moments of the vertical components dV.

$$dM' = x\ dV$$

$$= x\ dP \sin dx$$

$$= x\ r\ \omega\ m\ dx\ \frac{x}{r}$$

$$= \omega\ m\ x^2\ dx$$

$$M' = \int_0^{b/2} \omega\ m\ x^2\ dx = \omega\ m \left| \frac{x^3}{3} \right|_0^{b/2}$$

$$= \omega\ m\ \frac{b^3}{24} = \omega\ \frac{G_f}{g} = \frac{b^2}{24}$$

$$= 8.27 \times \frac{700}{9.81} \times \frac{20^2}{24} = 9825 \text{ mkg.}$$

The reaction R now becomes

$$R = \frac{9825}{6} = 1637.5 \text{ kg}$$

and the resulting spar stress is

$$H = \frac{6}{2.8} \times 1637.5 = 3500 \text{ kg}$$

Due to the opposite acting load, the horizontal stresses of the right and left wings are added,

$$2 \times 3500 = 7000 \text{ kg}$$

The total lateral stress finally becomes

$$S = 7000 + 887 = 7887 \text{ kg.}$$

It must be admitted that the calculated stress is large enough to justify the calculation of this case. Although it is possible for the stresses to be considerably smaller in many structural arrangements, or for the stresses to occur in structural members which must be amply dimensioned for other reasons, this complicated dependence on the static structure requires verification. I will refer to one more arrangement which is very often met with. Two points of the rear spar are often rigidly joined to the fuselage, and the front spar is held only by a strut at each of two other points. The lateral force, acting in the line of mass somewhere between the front and rear spar, produces a further rotational moment about the two points of support

of the rear spar. The result is a further considerable inequality in the stresses, to the disadvantage of individual structural elements.

Translation by Dwight M. Miner,
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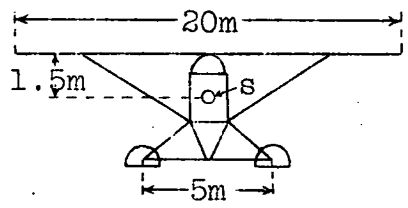

Fig.1

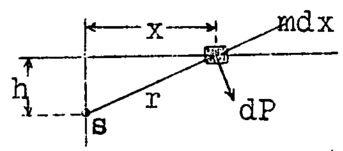

Fig.2

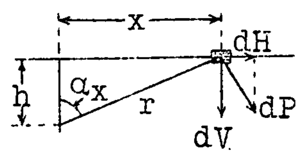

Fig.3

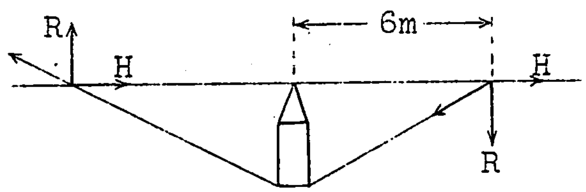

Fig.4